

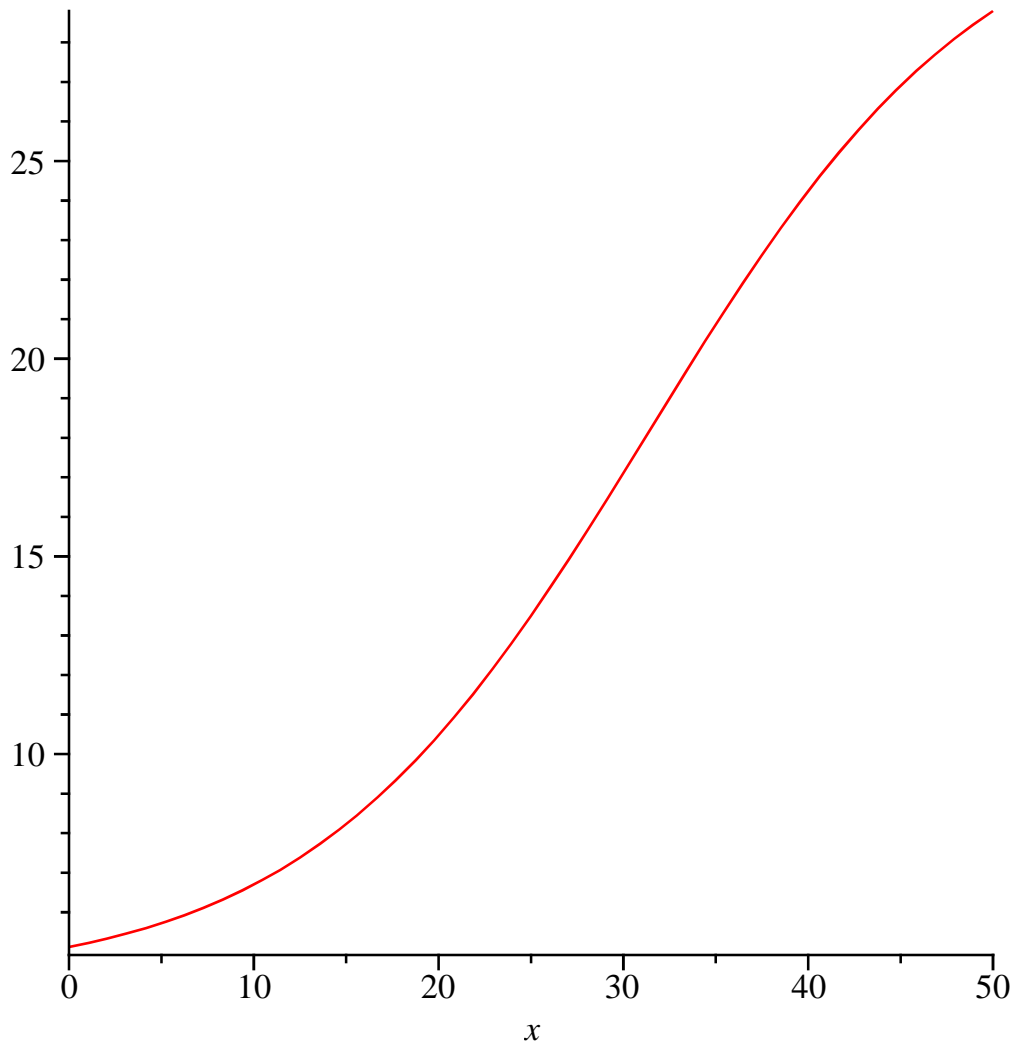
# Best-fit line Applied to Rager's method of Hop Utilization, Using Cubic Spline Interpolation

<http://realbeer.com/hops/FAQ.html>

[http://en.wikipedia.org/wiki/Spline\\_\(mathematics\)](http://en.wikipedia.org/wiki/Spline_(mathematics))

$$f := (x) \rightarrow 18.11 + \left( 13.86 \cdot \tanh\left(\frac{(x - 31.32)}{18.27}\right) \right)$$
$$x \rightarrow 18.11 + 13.86 \tanh\left(\frac{x - 31.32}{18.27}\right) \quad (1)$$

*plot(f(x), x=0..50)*



*f(0)*

5.12080637

(2)

*f(5)*

$$5.72164004 \quad (3)$$

$f(40)$

$$24.24038064 \quad (4)$$

$f(45)$

$$26.90297341 \quad (5)$$

$RagerData := [[5, 5], [10, 6], [15, 8], [20, 10.1], [25, 12.1], [30, 15.3], [35, 18.8], [40, 22.8], [45, 26.9]]$

$[[5, 5], [10, 6], [15, 8], [20, 10.1], [25, 12.1], [30, 15.3], [35, 18.8], [40, 22.8], [45, 26.9]] \quad (6)$

$with(CurveFitting) :$

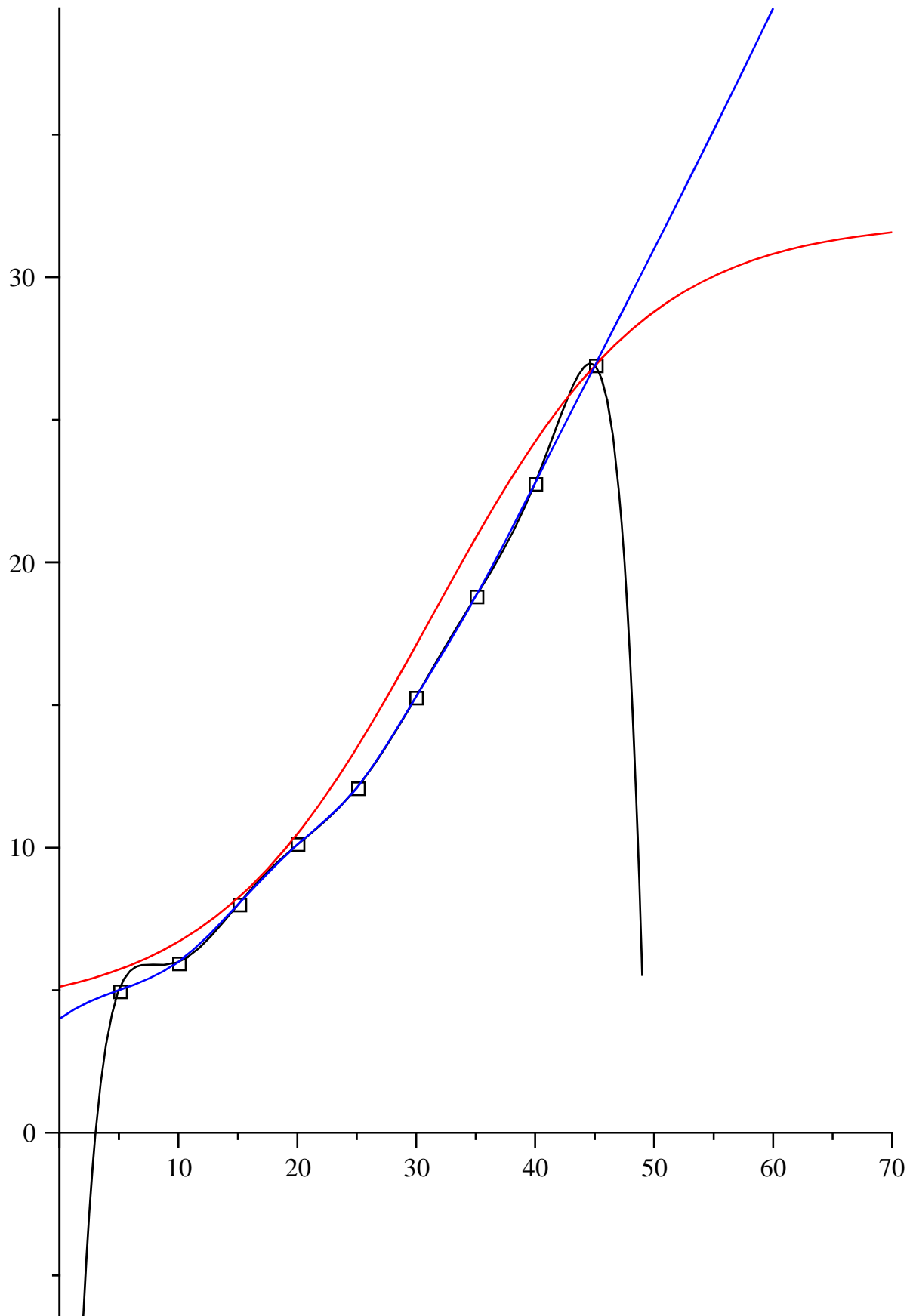
$cubfit := PolynomialInterpolation(RagerData, x);$

$$\begin{aligned} & -1.492063487 \cdot 10^{-9} x^8 + 2.977777768 \cdot 10^{-7} x^7 - 0.00002485555548 x^6 + 0.001125244441 x^5 \\ & - 0.02995347214 x^4 + 0.4750361099 x^3 - 4.321605148 x^2 + 20.58616662 x \\ & - 33.699999990 \end{aligned} \quad (7)$$

$plots[display]($

$plot(RagerData, style = point, color = black, symbol = BOX),$

$plot(cubfit, x = 2 ..49, color = black), plot(f(x), x = 0 ..70, color = red), plot(Spline(RagerData, x), x = 0 ..60, color = blue);$



*Spline(RagerData, x)*

$$\left\{ \begin{array}{ll}
 4.252899484 + 0.149420103100000006 x + 0.00202319587600000017 (x - 5)^3 & x < 10 \\
 2.988402062 + 0.301159793800000020 x + 0.0303479381443298932 (x - 10)^2 - 0.00211597938099999982 (x - 10)^3 & x < 15 \\
 1.310889176 + 0.445940721599999978 x - 0.00139175257731958734 (x - 15)^2 - 0.000759278350700000024 (x - 15)^3 & x < 20 \\
 2.598453608 + 0.375077319600000014 x - 0.0127809278350515462 (x - 20)^2 + 0.00355309278299999980 (x - 20)^3 & x < 25 \\
 -0.74375000 + 0.513750000000000040 x + 0.0405154639175257722 (x - 25)^2 - 0.00305309278400000022 (x - 25)^3 & x < 30 \\
 -5.39768041 + 0.689922680400000042 x - 0.00528092783505154738 (x - 30)^2 + 0.00145927835099999996 (x - 30)^3 & x < 35 \\
 -7.32957474 + 0.746559278400000026 x + 0.0166082474226804146 (x - 35)^2 - 0.00118402061899999996 (x - 35)^3 & x < 40 \\
 -10.15360825 + 0.823840206200000091 x - 0.00115206185567010338 (x - 40)^2 + 0.0000768041237299999924 (x - 40)^3 & otherwise
 \end{array} \right. \quad (8)$$

*with(CurveFitting) :*  
*Spline(RagerData, x);*

$$\left\{ \begin{array}{ll}
 4.252899484 + 0.149420103100000006 x + 0.00202319587600000017 (x - 5)^3 & x < 10 \\
 2.988402062 + 0.301159793800000020 x + 0.0303479381443298932 (x - 10)^2 - 0.00211597938099999982 (x - 10)^3 & x < 15 \\
 1.310889176 + 0.445940721599999978 x - 0.00139175257731958734 (x - 15)^2 - 0.000759278350700000024 (x - 15)^3 & x < 20 \\
 2.598453608 + 0.375077319600000014 x - 0.0127809278350515462 (x - 20)^2 + 0.00355309278299999980 (x - 20)^3 & x < 25 \\
 -0.74375000 + 0.513750000000000040 x + 0.0405154639175257722 (x - 25)^2 - 0.00305309278400000022 (x - 25)^3 & x < 30 \\
 -5.39768041 + 0.689922680400000042 x - 0.00528092783505154738 (x - 30)^2 + 0.00145927835099999996 (x - 30)^3 & x < 35 \\
 -7.32957474 + 0.746559278400000026 x + 0.0166082474226804146 (x - 35)^2 - 0.00118402061899999996 (x - 35)^3 & x < 40 \\
 -10.15360825 + 0.823840206200000091 x - 0.00115206185567010338 (x - 40)^2 + 0.0000768041237299999924 (x - 40)^3 & otherwise
 \end{array} \right. \quad (9)$$

simplify piecewise

$$\left\{ \begin{array}{ll}
 3.999999999 + 0.3011597938 x + 0.002023195876 x^3 - 0.03034793814 x^2 & x < 10 \\
 8.139175257 - 0.9405927834 x + 0.09382731957 x^2 - 0.002115979381 x^3 & x < 15 \\
 3.560309280 - 0.02481958780 x + 0.03277577320 x^2 - 0.0007592783507 x^3 & x < 20 \\
 -30.93865979 + 5.150025773 x - 0.2259664948 x^2 + 0.003553092783 x^3 & x < 25 \\
 72.28298970 - 7.236572166 x + 0.2694974227 x^2 - 0.003053092784 x^3 & x < 30 \\
 -49.55103094 + 4.946829898 x - 0.1366159794 x^2 + 0.001459278351 x^3 & x < 35 \\
 63.78041239 - 4.767293816 x + 0.1409304124 x^2 - 0.001184020619 x^3 & x < 40 \\
 -16.91237114 + 1.284664949 x - 0.01036855670 x^2 + 0.00007680412373 x^3 & 40 \leq x
 \end{array} \right.$$

$$\begin{aligned}
 g := (x) \rightarrow & \text{piecewise}(x < 10, 3.999999999 + 0.3011597938 x + 0.002023195876 x^3 + (-1) \\
 & \cdot 0.03034793814 x^2, x < 15, 8.139175257 + (-1) \cdot 0.9405927834 x + 0.09382731957 x^2 + (-1) \\
 & \cdot 0.002115979381 x^3, x < 20, 3.560309280 + (-1) \cdot 0.02481958780 x + 0.03277577320 x^2 + (-1) \\
 & \cdot 0.0007592783507 x^3, x < 25, -30.93865979 + 5.150025773 x + (-1) \cdot 0.2259664948 x^2 \\
 & + 0.003553092783 x^3, x < 30, 72.28298970 + (-1) \cdot 7.236572166 x + 0.2694974227 x^2 + (-1) \\
 & \cdot 0.003053092784 x^3, x < 35, -49.55103094 + 4.946829898 x + (-1) \cdot 0.1366159794 x^2 \\
 & + 0.001459278351 x^3, x < 40, 63.78041239 + (-1) \cdot 4.767293816 x + 0.1409304124 x^2 + (-1) \\
 & \cdot 0.001184020619 x^3, x \leq 45, -16.91237114 + 1.284664949 x + (-1) \cdot 0.01036855670 x^2 \\
 & + 0.00007680412373 x^3, x > 45, f(x) )
 \end{aligned}$$

$$\begin{aligned}
 x \rightarrow & \text{piecewise}(x < 10, 3.999999999 + 0.3011597938 x + 0.002023195876 x^3 + (-1) & (10) \\
 & \cdot 0.03034793814 x^2, x < 15, 8.139175257 + (-1) \cdot 0.9405927834 x + 0.09382731957 x^2 + (-1) \\
 & \cdot 0.002115979381 x^3, x < 20, 3.560309280 + (-1) \cdot 0.02481958780 x \\
 & + 0.03277577320 x^2 + (-1) \cdot 0.0007592783507 x^3, x < 25, -30.93865979 + 5.150025773 x \\
 & + (-1) \cdot 0.2259664948 x^2 + 0.003553092783 x^3, x < 30, 72.28298970 + (-1) \\
 & \cdot 7.236572166 x + 0.2694974227 x^2 + (-1) \cdot 0.003053092784 x^3, x < 35, -49.55103094 \\
 & + 4.946829898 x + (-1) \cdot 0.1366159794 x^2 + 0.001459278351 x^3, x < 40, 63.78041239 \\
 & + (-1) \cdot 4.767293816 x + 0.1409304124 x^2 + (-1) \cdot 0.001184020619 x^3, x \leq 45, \\
 & -16.91237114 + 1.284664949 x + (-1) \cdot 0.01036855670 x^2 + 0.00007680412373 x^3, 45 \\
 & < x, f(x) )
 \end{aligned}$$

evaluate procedure  $\rightarrow$

$$\left\{ \begin{array}{ll}
3.999999999 + 0.3011597938 x + 0.002023195876 x^3 - 0.03034793814 x^2 & x < 10 \\
8.139175257 - 0.9405927834 x + 0.09382731957 x^2 - 0.002115979381 x^3 & x < 15 \\
3.560309280 - 0.02481958780 x + 0.03277577320 x^2 - 0.0007592783507 x^3 & x < 20 \\
-30.93865979 + 5.150025773 x - 0.2259664948 x^2 + 0.003553092783 x^3 & x < 25 \\
72.28298970 - 7.236572166 x + 0.2694974227 x^2 - 0.003053092784 x^3 & x < 30 \\
-49.55103094 + 4.946829898 x - 0.1366159794 x^2 + 0.001459278351 x^3 & x < 35 \\
63.78041239 - 4.767293816 x + 0.1409304124 x^2 - 0.001184020619 x^3 & x < 40 \\
-16.91237114 + 1.284664949 x - 0.01036855670 x^2 + 0.00007680412373 x^3 & x \leq 45 \\
18.11 + 13.86 \tanh(0.05473453749 x - 1.714285714) & 45 < x
\end{array} \right. \quad (11)$$

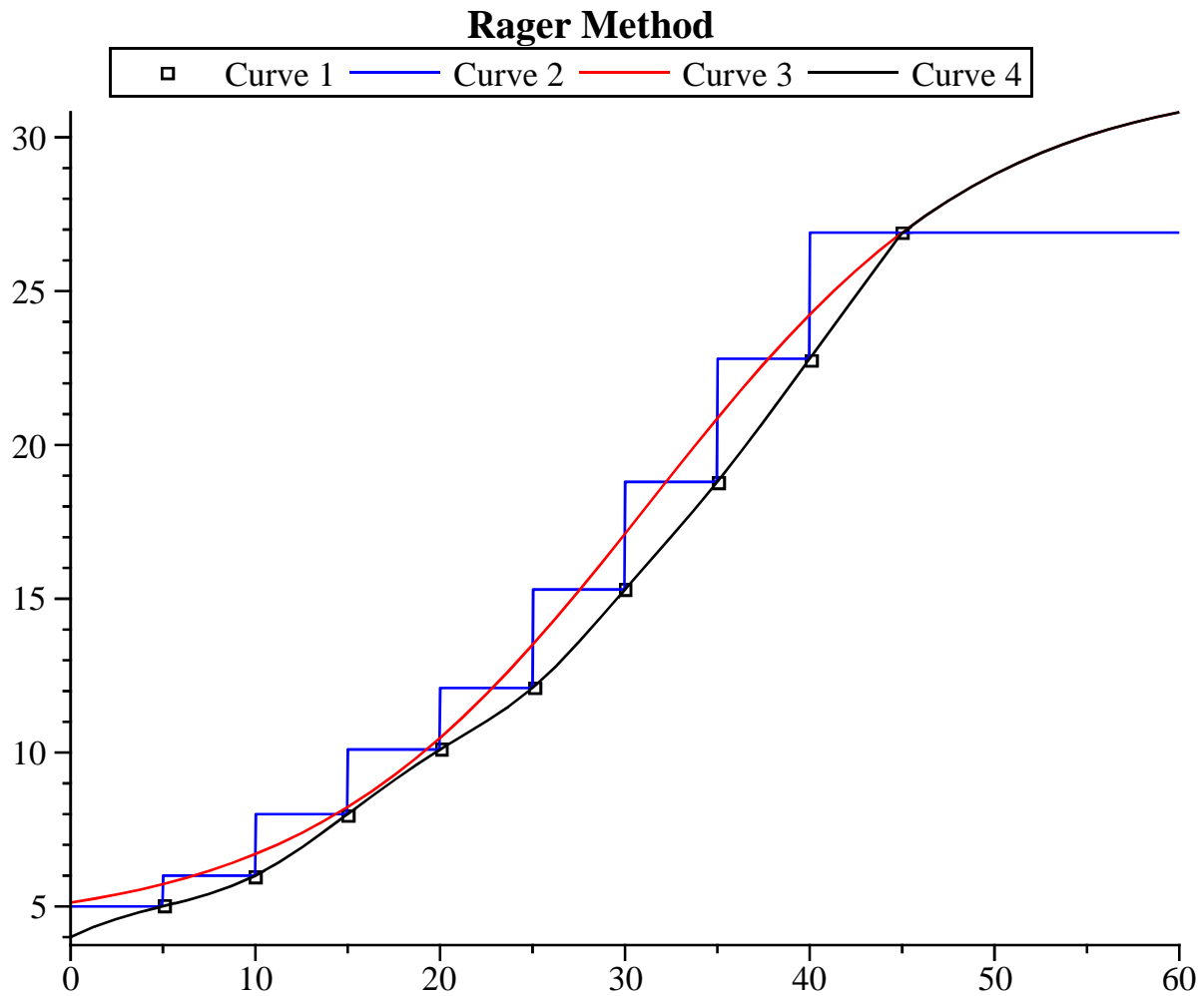
*RagerData*

$$[[5, 5], [10, 6], [15, 8], [20, 10.1], [25, 12.1], [30, 15.3], [35, 18.8], [40, 22.8], [45, 26.9]] \quad (12)$$

$$h := (x) \rightarrow \text{piecewise}(x < 5, 5, x < 10, 6, x < 15, 8, x < 20, 10.1, x < 25, 12.1, x < 30, 15.3, x < 35, 18.8, x < 40, 22.8, x \geq 40, 26.9)$$

$$x \rightarrow \text{piecewise}(x < 5, 5, x < 10, 6, x < 15, 8, x < 20, 10.1, x < 25, 12.1, x < 30, 15.3, x < 35, 18.8, x < 40, 22.8, 40 \leq x, 26.9) \quad (13)$$

```
plots[display](plot(RagerData, style = point, color = black, symbol = BOX),
plot(h(x), x = 0 ..60, color = blue),
plot(f(x), x = 0 ..60, color = red),
plot(g(x), x = 0 ..60, color = black),
);
```



$$\text{Utilization\%} := (x) \rightarrow 18.11 + \left( 13.86 \cdot \tanh\left( \frac{(x - 31.32)}{18.27} \right) \right)$$

$$x \rightarrow 18.11 + 13.86 \tanh\left( \frac{x - 31.32}{18.27} \right) \quad (14)$$

```
plots[display](plot(RagerData, style = point, color = black, symbol = BOX),
plot(h(x), x = 0 ..60, color = blue),
plot(f(x), x = 0 ..60, color = red),
plot(g(x), x = 0 ..60, color = black),
);
```

## Rager's Method

